2nd International Conference on Symmetry, Benasque, Spain

CLASSICAL ORTHOGONAL POLYNOMIALS

ORTHOGONALITY AND DUALITY

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Slides: www.rscosan.com/talks.html

OUTLINE

- We introduce the basics.
- We are going to describe the relations among the different families in the different schemes.

THE BASICS

HYPERGEOMETRIC FUNCTIONS

$$ {}_rF_s\left(\begin{array}{cccc} a_1, & a_2, & \ldots, & a_r \\ b_1, & b_2, & \ldots, & b_s \end{array}; z\right) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_r)_k}{(b_1)_k (b_2)_k \cdots (b_s)_k} \frac{z^k}{k!} $$

- Depending on 'where' we insert the variable in this special function it fulfills a differential, or difference equation.

$$ p(x)\frac{d^2}{dx^2} + q(x)\frac{d}{dx} $$

$$ (q(x)+p(x))\mathfrak{s}_1 - (q(x)+2p(x))\mathfrak{s}_0 + p(x)\mathfrak{s}_{-1} $$

BASIC HYPERGEOMETRIC FUNCTIONS

$$ {}_r\phi_s\left(\begin{array}{cccc} a_1, & a_2, & \dots, & a_r \\ b_1, & b_2, & \dots, & b_s \end{array} \middle| q; z\right) = \sum_{k=0}^{\infty} \frac{(a_1;q)_k (a_2;q)_k \cdots (a_r;q)_k}{(b_1;q)_k (b_2;q)_k \cdots (b_s;q)_k} (-1)^{(1+s-r)} q^{(1+s-r)\binom{k}{2}} \frac{z^k}{(q;q)_k} $$

The non uniform lattice is $x(s) = c_1 q^s + c_2 q^{-s} + c_3$

$$ A(x(s))h_q + B(x(s))h_1 + C(x(s))h_{1/q} $$

RECURRENCE RELATION

$$xp_n(x) = \alpha_n p_{n+1}(x) + \beta_n p_n(x) + \gamma_n p_{n-1}(x),$$

Result: If there exists a positive integer N so that

$$\gamma_N = 0.$$

Then, we can define a new inner product

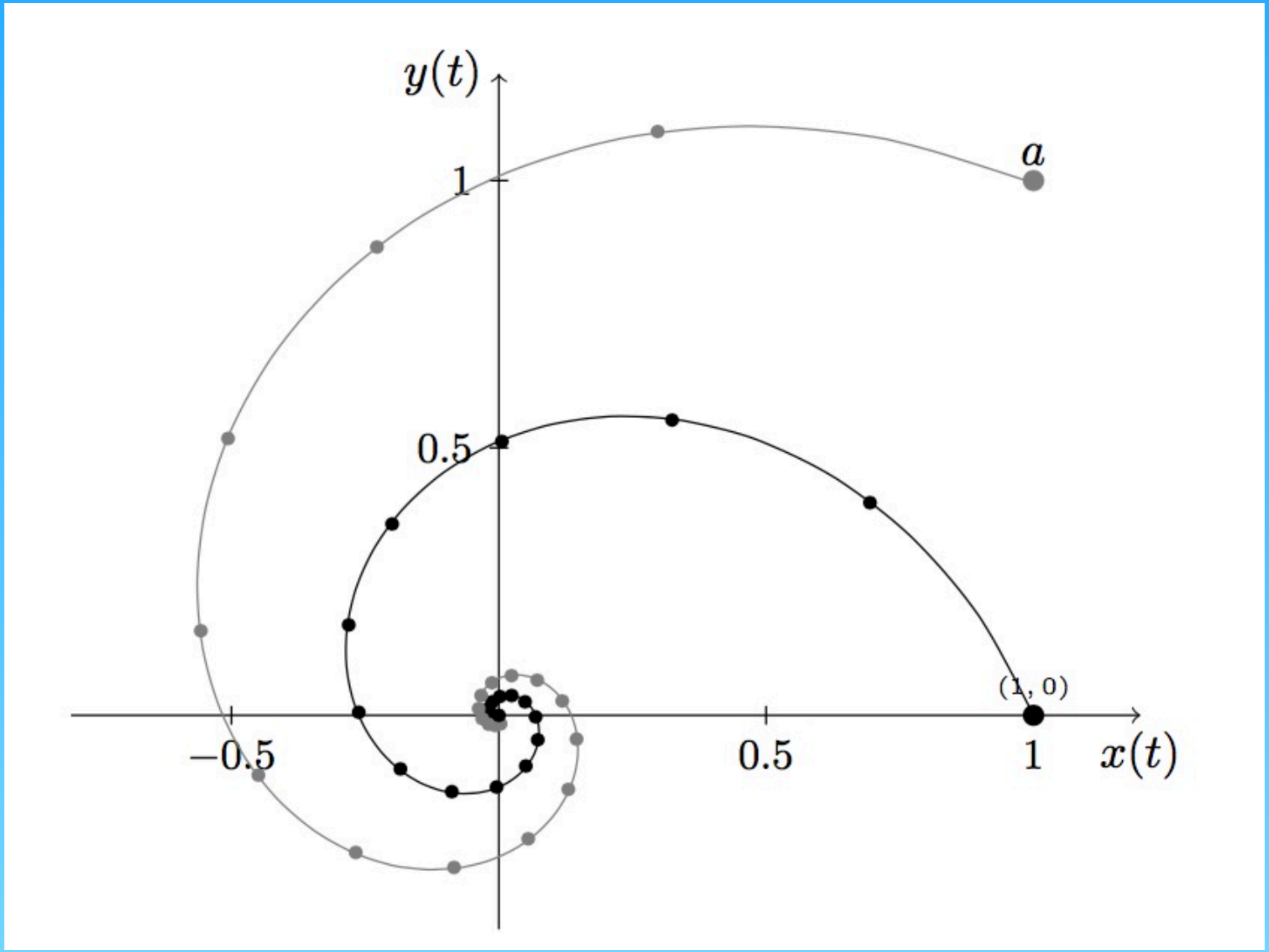
$$\langle f, g \rangle = \langle f, g \rangle_d + \langle \mathcal{D}^N f, \mathcal{D}^N g \rangle_c$$

Such that, the sequence $\{p_n(x)\}_{n=0}^{M}$, $M > N$

is orthogonal with respect to.

ORTHOGONALITY

$$xp_n(x) = \alpha_n p_{n+1}(x) + \beta_n p_n(x) + \gamma_n p_{n-1}(x),$$



DUALITY

Definition: Given two sets of nonnegative integers

$$U, V \subseteq \mathbb{N}_0$$

We say that **the two sequences of polynomials**

$$(p_u)_{u\in U}, \ (q_v)_{v\in V}$$

are **dual** if there exists a couple of sequences of numbers

$$(\xi_u)_{u\in U}, \ (\zeta_v)_{v\in V}$$

such that $$\xi_u p_u(v) = \zeta_v p_v(u), \quad u \in U, \, v \in V.$$

THE SCHEMES

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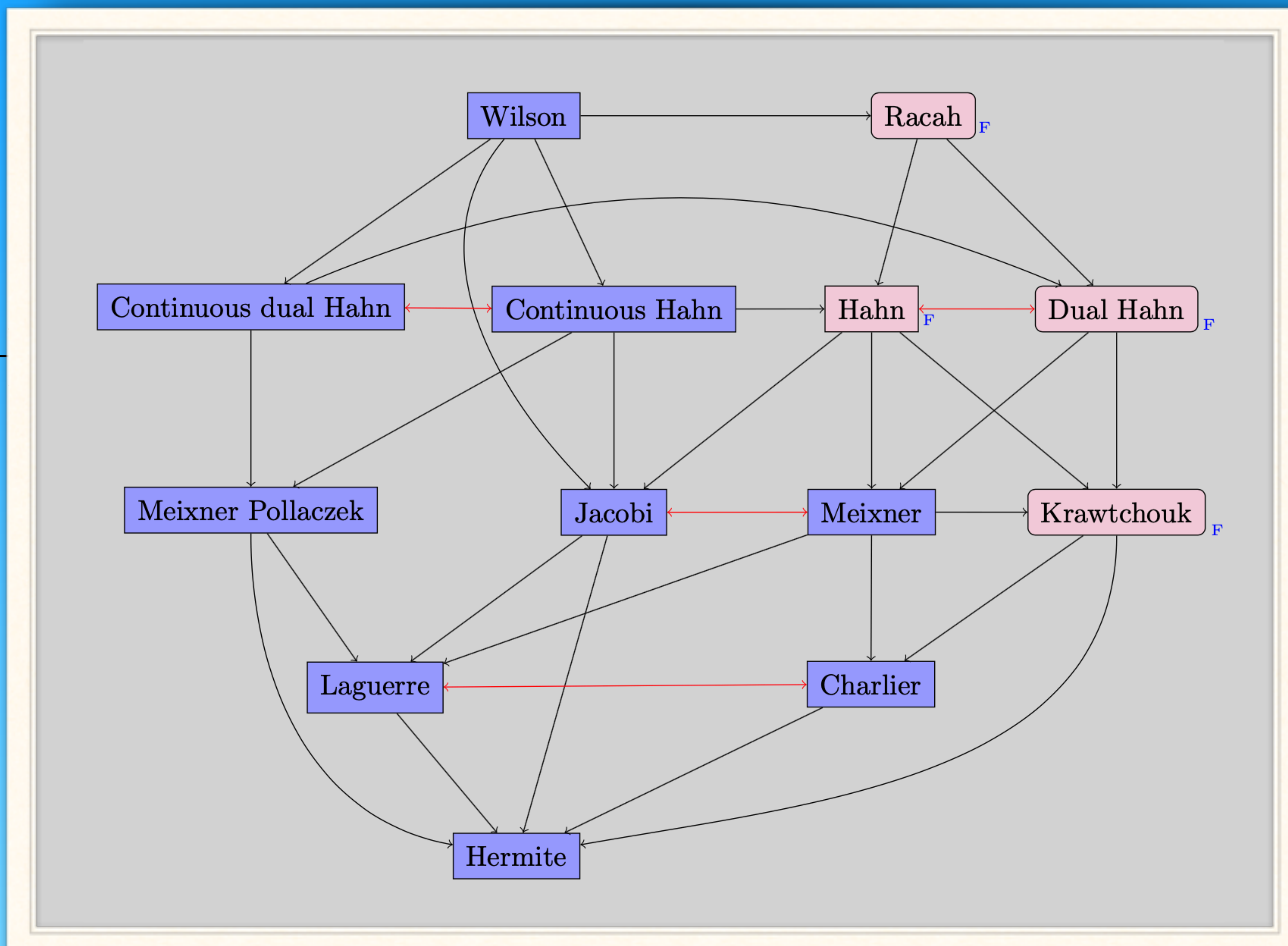
Wilson
Racah
F
Continuous dual Hahn
Continuous Hahn
Hahn
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Meixner Pollaczek
Jacobi
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Krawtchouk
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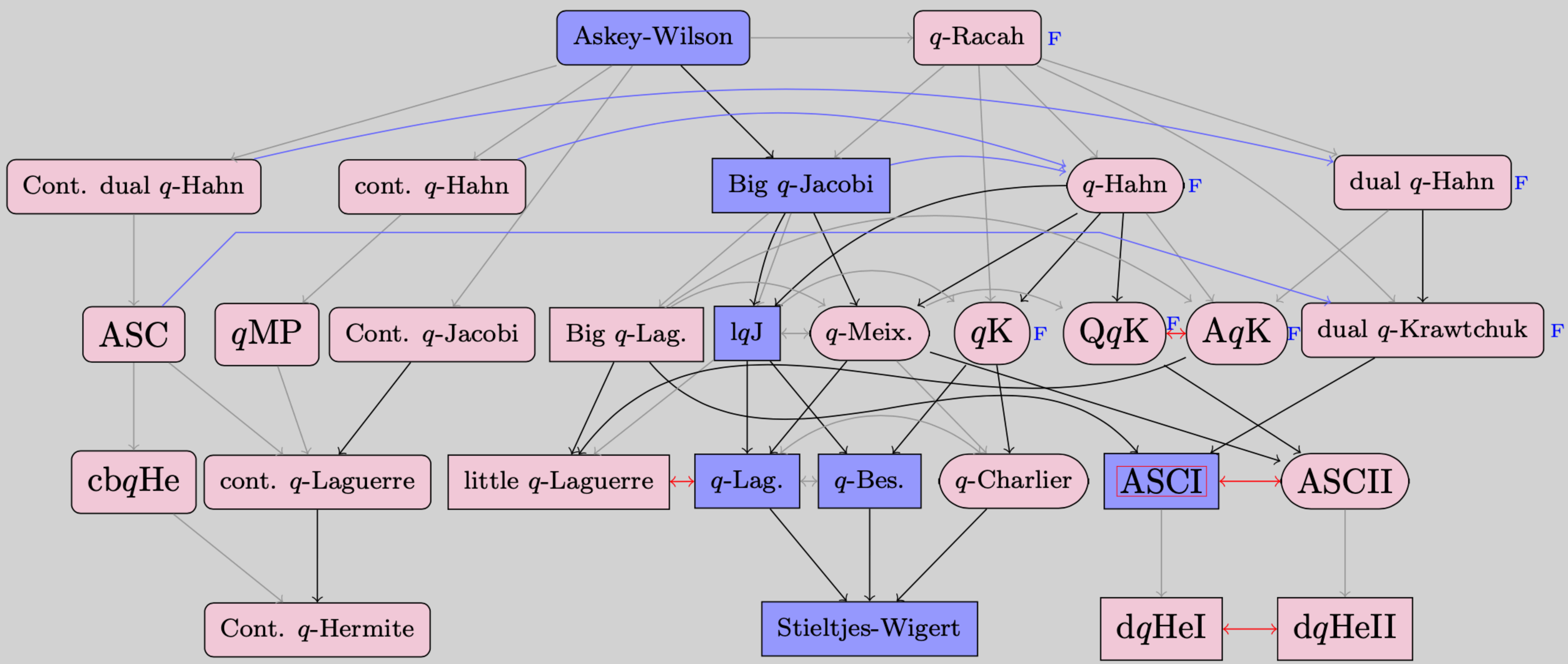
Laguerre
Charlier
Hermite


Askey-Wilson
q-Racah
F
Cont. dual q-Hahn
cont. q-Hahn
Big q-Jacobi
q-Hahn
F
dual q-Hahn
F
ASC
qMP
Cont. q-Jacobi
Big q-Lag.
lqJ
q-Meix.
qK
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QqK
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AqK
F
dual q-Krawtchuk
F
cbqHe
cont. q-Laguerre
little q-Laguerre
q-Lag.
q-Bes.
q-Charlier
ASCI
ASCII
Cont. q-Hermite
Stieltjes-Wigert
dqHeI
dqHeII

Thank you for your attention